Low-Complexity Iterative Sinusoidal Parameter Estimation

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Context

Context: Approximating a signal as a sum of sinusoids

- Audio compression
- Audio processing

Problem:

- Estimating sinusoidal parameters is a non-linear problem
- Non-linear problems are computationally expensive
- Must often be done in real-time with few resources

Solution

- Linearising the problem as much as possible
- Using an iterative solver

Sinusoidal Parameters

A sinusoid is defined as

- Amplitude
- Phase
- Frequency

(Amplitude, Phase:) Can be estimated linearly (e.g. FFT)

(Frequency:) Non-linear

We consider a fourth parameter

- Linear amplitude modulation

$$\tilde{x}(n) = h(n) \sum_{k=1}^{N} \left(A_k + nA_k^{'} \right) \cdot \cos\left(\left(\theta_k + \Delta\theta_k \right) n + \phi_k \right)$$

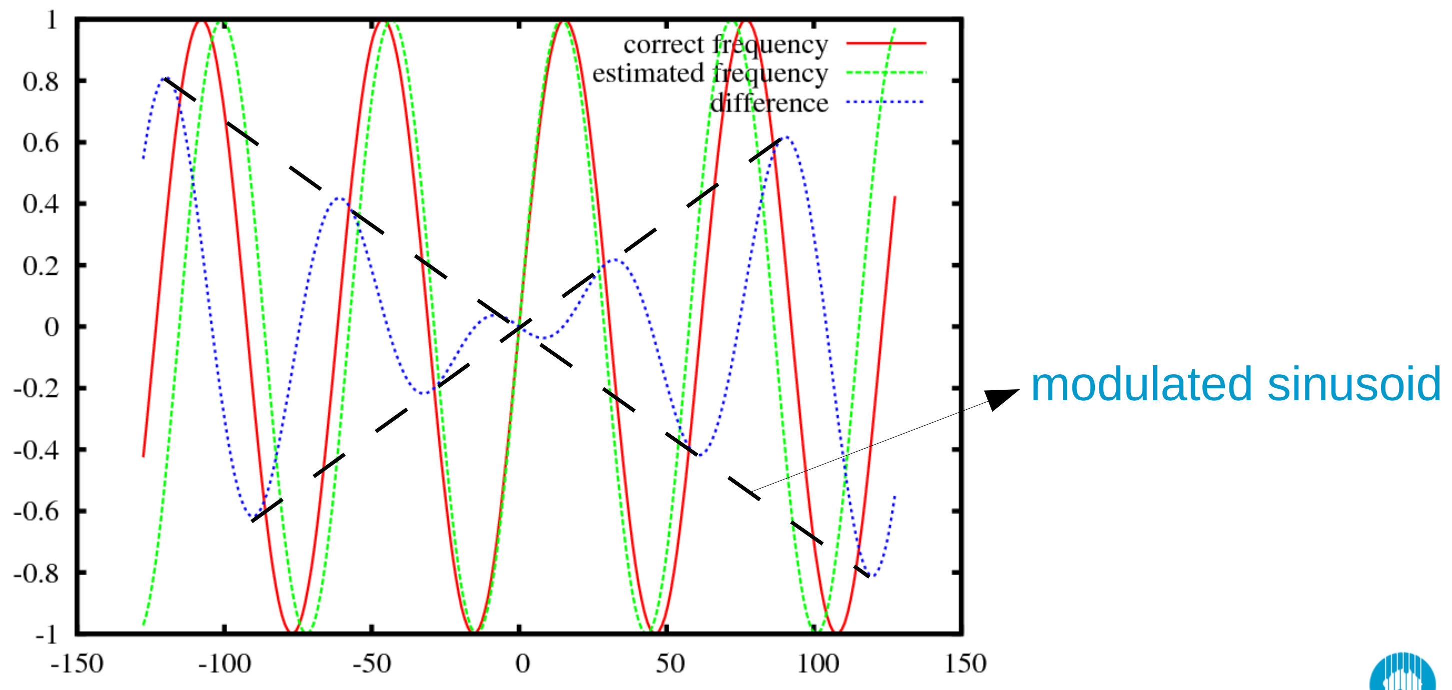
Workaround: Linearisation

Hypothesis #1: We have an initial estimate of frequencies

- Obtained though a lower resolution FFT
- From previous time frame

Hypothesis #2: The error on the estimate is small

Result: Frequency behaves *almost* linearly



Linear System

Any sinusoid can be expressed as the sum of 4 basis functions

$$\tilde{x}(n) = h(n) \sum_{k=1}^{N} c_k \cos\theta_k n + s_k \sin\theta_k n + d_k n \cos\theta_k n + t_k n \sin\theta_k n$$

Parameters are (neglecting 2nd order terms):

$$A_k = \sqrt{c_k^2 + s_k^2}$$

$$\phi_k = \arg(c_k - \jmath s_k)$$

$$A_k^{'} = \frac{d_k c_k + s_k t_k}{A_k}$$

$$\Delta\theta_k = \frac{d_k s_k - t_k c_k}{A_k^2}$$

Linear Solver

Direct solver is $O(LN^2)$

Iterative method: Gauss-Seidel in $O(LN)$

- Basis is nearly orthogonal, guaranteed convergence
- Successive projections of the error on the basis functions
- First cos/sin terms, then modulated terms (faster convergence)

for all iteration i=1…M **do**
  for all sinusoid component $k = 1 \ldots 4N$ **do**
    $\Delta w_k^{(i)} \leftarrow \mathbf{a}_k^T \mathbf{e}$
    $\mathbf{e} \leftarrow \mathbf{e} - \mathbf{a}_k \Delta w_k^{(i)}$
    $w_k^{(i)} \leftarrow w_k^{(i-1)} + \Delta w_k^{(i)}$
  end for
end for

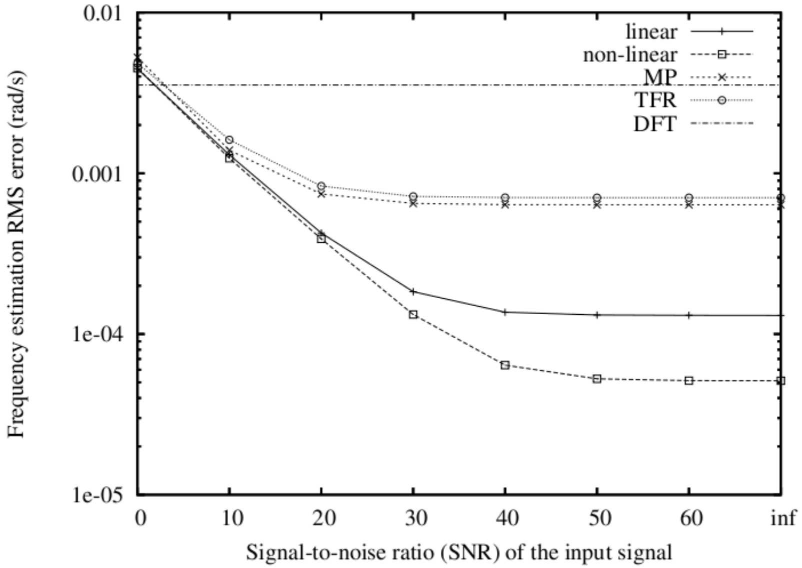
Non-Linear Solver

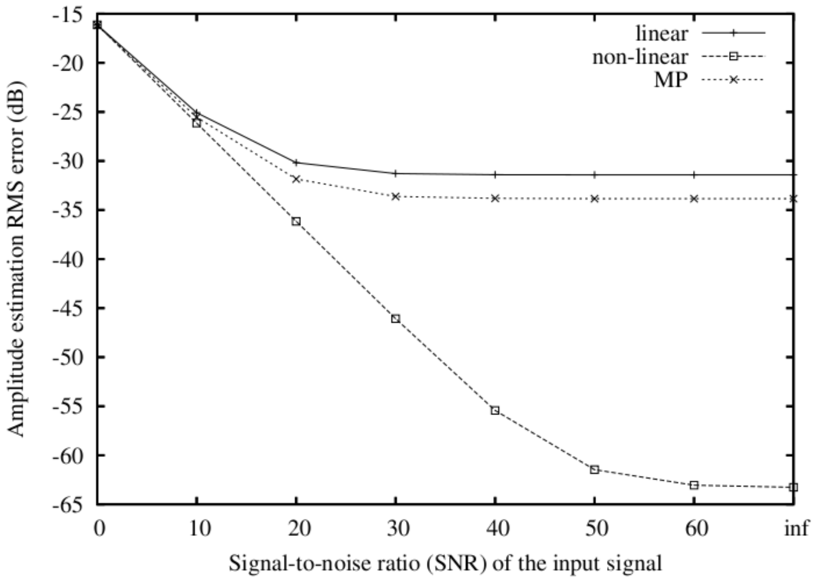
- Linear solution is imperfect when frequency error is too large
- Non-linear solver adjusts the frequency for every iteration
  - Compute one linear iteration
  - Compute sinusoid parameters (including new frequency)
  - Recompute the error based on the non-linear parameters
  - Goto 1)
- Complexity
  - Only a small increase compared to the linear solution:
    - Need to re-compute the basis functions
    - Slightly longer to converge

Results

Frequency and amplitude accuracy (5 chirps with noise)

- Linear solution
- Non-linear solution
- Matching pursuit
- Time-frequency reassignment
- DFT

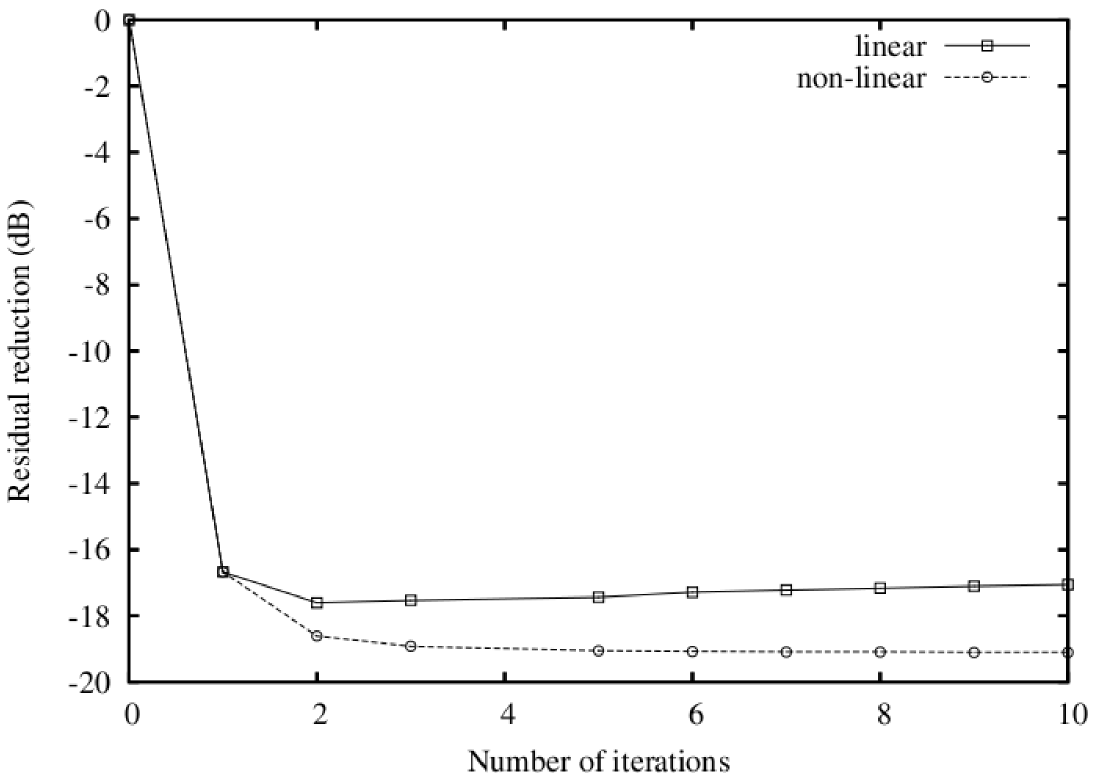




Convergence

Convergence on a music signal

- Linear solution requires 2 iterations
- Non-linear solution requires 3 iterations



Complexity

L: Length of the input data (256)

M: Number of iterations (2 for linear, 3 for non-linear)

N: Number of sinusoids (20)

P: Matching pursuit oversampling (32)

Algorithm	Complexity	Typical (Mflops)
MP (slow)	$4LN^2P$	3,300
MP (FFT)	$\frac{5}{2}LNP\log_2 LP$	1,300
linear (18)	$64N^3 + 32LN^2$	900
non-linear ([5])	$O\left(N^4 + LN^2\right)$	>500*
linear (proposed)	$(8M+5)\,LN$	27
non-linear (prop.)	$(17M-4)\,LN$	60

Conclusion

A low-complexity method for estimating sinusoid parameters

- Linearisation of the estimation problem
- Iterative solution (Gauss-Seidel)
- Optional non-linear solution

Reduces complexity by 1-2 orders of magnitude compared to other algorithms

Future work

- Improve initial frequency estimates
- Extend to the estimation of frequency modulation

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